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LETTER TO THE EDITOR

Universal finite-size effects in the rate of growth processes

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Abstract. Kinetic roughening of the growing surface generates universal finite-size corrections in the growth rate of films and crystals. For thin films the correction scales with the film thickness h as $h^{-\alpha_\perp}$; for thick films it scales with the substrate size L as $L^{-\alpha_\parallel}$, where $\alpha_\parallel = 2(1-\zeta)$ and $\alpha_\perp = 2(1-\zeta)/z$ in terms of the kinetic roughening exponents ζ and z. For ballistic deposits this implies a similar correction in the density. The coefficient of the correction is proportional to the KPZ coupling constant λ. For one-dimensional substrates $\alpha_\parallel = 1$ and $\alpha_\perp = \frac{2}{3}$. These predictions are corroborated by computer simulations of growth and deposition on one- and two-dimensional substrates, and by exact results for one-dimensional models. Different exponents apply in the weak results for one-dimensional models. Different exponents apply in the weak coupling regime and at kinetic roughening transitions.

Transients in crystal growth rates reflect the evolution of the surface morphology, thereby providing insight into the growth mechanism [1]. Similarly in the growth of whiskers [2] and lamellar polymer crystals [3-6] it is important to understand the dependence of growth rates on the crystal size. In the present letter we study transient and size effects related to the kinetic roughening [7] of the growing surface. It has only recently been realized [8] that surface roughening in growth processes involves long wavelength undulations [9] and therefore is a universal phenomenon independent of microscopic details.

Consider for example a film grown from a perfectly flat substrate of linear size L. The growth process starts at time $t = 0$. To leading order in t the film thickness $\bar{h}$ is proportional to t. Hence the growth rate $G(L, t) = \mathrm{d}\bar{h}/\mathrm{d}t$ tends to some limiting value G_0 for $t, L \to \infty$. We are going to show that the finite size correction

$$\Delta G(L, t) = G(L, t) - G_0 \tag{1}$$

has a power law decay

$$\Delta G \sim \begin{cases} -\lambda L^{-\alpha_\parallel} & \text{for } t \gg L^z \\ -\lambda t^{-\alpha_\perp} & \text{for } t \ll L^z \end{cases} \tag{2}$$

where the exponents $\alpha_\parallel$ and $\alpha_\perp$ depend only on the dimensionality d of the substrate, and the coefficient λ is related to the inclination dependence of the macroscopic growth

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rate G_0. The dynamic exponent z which determines the crossover time scale $t_c \sim L^z$ in (2) describes the lateral spreading of surface fluctuations in the growth process [7, 8, 10] (e.g. $z=2$ for diffusive dynamics). For a growing crystal of radius R we have $L \sim t \sim R$. Since $z>1$, this implies that $t \ll L^z$ and the relevant exponent is $\alpha_\perp$.

Throughout the letter periodic boundary conditions are assumed in the d substrate directions. More realistic (free) boundary conditions affect only the amplitude of ΔG, but not the values of the exponents $\alpha_\parallel$ and $\alpha_\perp$. An explicit one-dimensional example is given below. In the presence of edges a growing film develops size-dependent corrections to the (flat) macroscopic shape [11] which give rise to additional corrections in the growth rate.

Our general results are derived for the continuum model of Kardar, Parisi and Zhang (KPZ) [8]. On a mesoscopic scale the local film thickness above a substrate point $\boldsymbol{x}=(x_1, \ldots, x_d)$, at time t, is described by a function $h(\boldsymbol{x}, t)$ which evolves stochastically according to

$$\frac{\partial}{\partial t} h(\boldsymbol{x}, t) = g_b + \frac{\lambda}{2}(\nabla h)^2 + \nu \nabla^2 h + \eta(\boldsymbol{x}, t). \tag{3}$$

Here g_b is the 'bare' growth rate in the absence of fluctuations. The nonlinear term reflects the inclination dependence [7, 8, 12, 13] of the local growth rate. The last two terms on the right-hand side represent the fluctuations in the growth process. The random flux $\eta(\boldsymbol{x}, t)$ is taken to have zero mean and short range correlations in space and time. The full growth rate of the infinite film is obtained by averaging (3) over the noise η. Since the average surface profile is flat, $\langle \nabla^2 h \rangle = 0$. Thus

$$G_0 = \langle \partial h / \partial t \rangle_\infty = g_b + \frac{\lambda}{2} \langle (\nabla h)^2 \rangle_\infty \tag{4}$$

which shows how the bare growth rate is renormalized by fluctuations in the local surface inclination. The significance of the coefficient λ can be seen by imposing an infinitesimal tilt, $h(\boldsymbol{x}, t) \to h(\boldsymbol{x}, t) + \boldsymbol{u} \cdot \boldsymbol{x}$, $|\boldsymbol{u}| \ll 1$. Noting that $\langle \nabla h \rangle = 0$, we find that the change induced in G_0 is $\frac{1}{2}\lambda \boldsymbol{u}^2$ and hence [7, 12, 13]

$$\lambda = \frac{1}{d} \sum_{j=1}^{d} \frac{\partial^2}{\partial u_j^2} G_0 \Big|_{\boldsymbol{u}=\boldsymbol{0}}. \tag{5}$$

In a finite system the statistical average on the right-hand side of (4) is modified: the long wavelength fluctuations are cut off and $\langle (\nabla h)^2 \rangle$ is reduced. We conclude that the size correction to the growth rate is given by

$$\Delta G(L, t) = -\frac{\lambda}{2} (\langle (\nabla h)^2 \rangle_\infty - \langle (\nabla h)^2 \rangle_{t,L}). \tag{6}$$

Since the quantity in round brackets is positive, the sign of the correction is determined by the sign of λ. The constrained average is conveniently evaluated in Fourier space,

$$\langle (\nabla h)^2 \rangle_{t,L} = \sum_{\boldsymbol{q} \neq \boldsymbol{0}} \boldsymbol{q}^2 \langle |\hat{h}(\boldsymbol{q}, t)|^2 \rangle \tag{7}$$

where the sum is over all non-zero modes in the hypercube L^d, and $\hat{h}(\boldsymbol{q}, t)$ denotes the Fourier transform of $h(\boldsymbol{x}, t)$. For small $q = |\boldsymbol{q}|$ and large t the average Fourier amplitudes attain the scaling form [7, 8, 10]

$$\langle |\hat{h}(\boldsymbol{q}, t)|^2 \rangle \approx L^{-d} q^{-(d+2\zeta)} f(q^z t). \tag{8}$$

Here the wandering exponent [14] ζ describes the scaling of the amplitude of transverse excursions $\xi_\perp$ of the surface on a lateral length scale $\xi_\parallel$, $\xi_\perp \sim \xi_\parallel^\zeta$. The scaling function f approaches a constant for $t \gg q^{-z}$ and vanishes for $t \ll q^z$. Inserting (8) into (7) it follows that the leading corrections to the sum are determined by the large-scale spatial cut-off L if $t \gg L^z$ and by the temporal cut-off $t^{1/z}$ if $t \ll L^z$, as expressed in (2), and the exponents are identified as

$$\alpha_\parallel = 2(1-\zeta) \qquad \alpha_\perp = 2(1-\zeta)/z. \tag{9}$$

The kinetic roughening exponents ζ and z are known exactly only for one-dimensional substrates, where [8] $\zeta = \frac{1}{2}$, $z = \frac{3}{2}$ and hence

$$\alpha_\parallel = 1 \qquad \alpha_\perp = \tfrac{2}{3} \qquad (d=1). \tag{10}$$

For two-dimensional substrates accurate numerical estimates [15] indicate that $\zeta \simeq 0.38 \pm 0.01$, $z \simeq 1.62 \pm 0.02$, leading to

$$\alpha_\parallel \simeq 1.25 \bullet 0.02 \qquad \alpha_\perp \simeq 0.77 \pm 0.02 \qquad (d=2). \tag{11}$$

Moreover the exact scaling relation [8, 16] $\zeta + z = 2$ implies

$$1/\alpha_\perp - 1/\alpha_\parallel = \tfrac{1}{2} \tag{12}$$

for all d.

In order to test these predictions we have carried out large-scale simulations of a variety of growth models on one- and two-dimensional substrates. In figure 1 we show results for the stationary ($t \to \infty$) growth rate in two dimensions. To extract the exponent $\alpha_\parallel$ we use tthe fact that $G(2L) - G(L)$ scales as $L^{-\alpha_\parallel}$ asymptotically. For small L there are systematic deviations from the power law due to higher-order terms in $1/L$, while for large L statistical uncertainties dominate. Similar restrictions of the scaling region arise in the transient case $L \to \infty$ (see figure 2). The systematic error can be assessed by estimating the exponents also from the inverse growth rate G^{-1}. The exponents presented in table 1 were obtained from the data in the scaling region, and the error estimates contain a combination of systematic and statistical uncertainties. The agreement with the predictions (10) and (11) is satisfying in all cases.

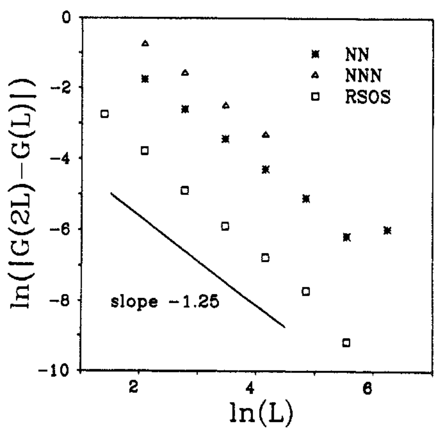

Figure 1. Size-dependent growth rate $G(L)$ for three models of growth on two-dimensional substrates: ballistic deposition with nearest-(NN) and next-nearest-(NNN) neighbour sticking [17, 18], and a restricted solid-on-solid (RSOS) model [19]. The power law $|G(2L) - G(L)| \sim L^{-1.25}$ is included to guide the eye.

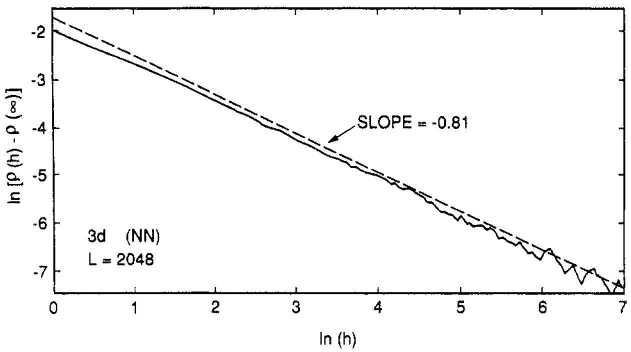

Figure 2. Deposit density ρ as a function of film thickness h for ballistic deposition with nearest-neighbour sticking on a substrate of 2048 × 2048 lattice sites. The asymptotic density $\rho(\infty) \simeq 0.3000$ was estimated by extrapolating the finite-size data shown in figure 1. The deposit density is inversely proportional to the growth rate.

Table 1. Numerical results for the finite-size correction exponents $\alpha_{\parallel}$ and $\alpha_{\perp}$. The models which were simulated are ballistic deposition with nearest-(NN) and next-nearest-(NNN) neighbour sticking [17, 18], a restricted solid-on-solid model [19], and the single step model [18]. The maximum substrate size for the determination of $\alpha_{\parallel}$ was $L = 1024$ (ballistic NN, $d = 1$), $L = 512$ (ballistic NN, $d = 2$), $L = 256$ (ballistic NNN, $d = 1$, and restricted SOS, $d = 2$), and $L = 128$ (ballistic NNN, $d = 2$ and restricted SOS, $d = 1$). The result (*) for the one-dimensional single step model is exact. The exponent $\alpha_{\perp}$ was obtained from several simulations on lattices of size $L = 2^{21}$ for $d = 1$, and $L = 2048$ (ballistic deposition) and $L = 1024$ (restricted SOS) for $d = 2$.

Model ($d = 1$)	$\alpha_{\parallel}$	$\alpha_{\perp}$
Ballistic NN	1.04 ± 0.05	0.70 ± 0.02
Ballistic NNN	1.01 ± 0.03	0.68 ± 0.03
Restricted SOS	0.98 ± 0.05	0.68 ± 0.01
Single step	1*	0.71 ± 0.02
Model ($d = 2$)	$\alpha_{\parallel}$	$\alpha_{\perp}$
Ballistic NN	1.28 ± 0.04	0.78 ± 0.04
Ballistic NNN	1.28 ± 0.08	0.83 ± 0.03
Restricted SOS	1.32 ± 0.09	0.81

The exponent $\alpha_{\parallel}$ was previously determined in simulations of spatially correlated ballistic deposition on one- and two-dimensional substrates [20]. In that case the roughening exponents ζ and z show a non-trivial dependence on the power law decay of the noise correlations [8]. The measured values [20] of $\alpha_{\parallel}$ and ζ are consistent with the scaling relation (9) both in one and two dimensions.

In the following we apply our results to some specialized situations.

(i) *Ballistic deposition.* For ballistic deposition processes the growth rate G is inversely proportional to the deposit density ρ, $\rho = J/G$ where J is the deposition flux [12]. The finite size correction to the deposit density is then $\Delta\rho \approx -(J/G_0^2)\Delta G$. Since $\lambda > 0$ for ballistic deposition [12], $\Delta\rho > 0$. Numerical results for ballistic deposition onto a two-dimensional substrate are shown in figure 2. Such power law density corrections were observed in several early studies of ballistic aggregation [21] and deposition [17]. The results are in rough agreement with our predictions, but at the time it was concluded that the corrections are non-universal. In view of the simulation results presented in table 1 it is clear that this apparent non-universality is due to additional corrections to scaling, which are known [22] to be strongly model dependent.

(ii) *$d = 1$.* In the one-dimensional case exact results are available for the size dependence of the stationary ($t \to \infty$) growth rate. These solutions rely on the fact [8, 23] that the surface gradient has short correlations in $d = 1$. This implies that the terms in the sum (7) are independent of q, $q^2\langle|\hat{h}(q)|^2\rangle = \chi$, where χ denotes the compressibility of the 'gas' of surface steps [7, 24]. Since the sum in (7) extends over $L-1$ of a total of L modes, we have

$$\Delta G(L) = -\frac{\lambda\chi}{2L} \qquad (d=1). \tag{13}$$

This expression is easily verified for the single step model [18], which has nearest-neighbour height differences $\sigma_j = h(j) - h(j-1) = \pm 1$, $j = 1, \ldots, L$ with periodic boundary conditions. In the steady state the σ_j at different sites are independent. The growth rate is equal to the probability to find a growth site (a local surface minimum), i.e. a pair of sites with $\sigma_j = -1$, $\sigma_{j+1} = 1$. For a surface of inclination $u = N/L$ there are $(L+N)/2$ sites with $\sigma_j = 1$ and $(L-N)/2$ sites with $\sigma_j = -1$. Hence the growth rate is

$$G(L) = \frac{1}{4}\frac{(L-N)(L+N)}{L(L-1)} = \frac{1}{4}(1-u^2)(1-1/L)^{-1}. \tag{14}$$

For $L \to \infty$, $G_0(u) = \frac{1}{4}(1-u^2)$ and $\lambda = -\frac{1}{2}$ according to (5). The compressibility is $\chi(u) = 1 - u^2$, so (13) gives $\Delta G = (1-u^2)/4L$ in agreement with (14).

A similar calculation can be done for the one-dimensional polynuclear growth model [3-5, 9] with nucleation rate Γ and step velocity c. In that case the macroscopic parameters entering (13) are [24] $\lambda = \sqrt{c^3/2\Gamma}$ and $\chi = \sqrt{2\Gamma/c}$, and thus $\Delta G(L) = -c/2L$ in agreement with the exact expression [5, 6] for $G(L)$. It is interesting to note that Frank's [4] approximate solution yields a leading correction term $\sim 1/L^2$ and hence appears to neglect [6] the effect of kinetic roughening.

We have also investigated the effect of free boundary conditions for the one-dimensional single step model [25]. Particles are added to the boundary sites $j = 1$ (L) at unit rate whenever $\sigma_2 = 1$ $(\sigma_L = -1)$. This dynamics leads to a horizontal surface, irrespective of the substrate inclination. The stationary height profile is concave ($h'' > 0$) and the growth rate is given by $G(L) = \frac{1}{4}(1 - 3/2L)^{-1}$. As expected, the boundary conditions merely change the amplitude of ΔG.

(iii) *Weak coupling.* In substrate dimensions $d \geq 3$ the KPZ equation (3) shows a phase transition at a critical value λ_c of the nonlinearity coefficient [15, 26]. In the weak coupling phase $|\lambda| < \lambda_c$ the nonlinearity is irrelevant and the scaling form (8) holds with the exponents obtained from the linearized version of (3), $\zeta = (2-d)/2$ and $z = 2$. Hence the size correction exponents are

$$\alpha_\parallel = d \qquad \alpha_\perp = d/2 \qquad \text{(weak coupling)}. \tag{15}$$

The result $\alpha_\parallel = d$ is typical for equilibrium systems, where finite-size corrections to canonical expectations values are of the order $(\text{volume})^{-1}$. In the limit $d \to \infty$ (15) indicates that the power law corrections (2) become exponential in the weak coupling phase. This is consistent with rigorous results obtained by Derrida and Spohn [27] on the Cayley tree, which is thought to represent the high-dimensionality limit. They computed the temporal finite size corrections, finding an exponential decay in the weak coupling phase and a $1/t$ decay in the strong coupling phase. Thus the $d \to \infty$ limit of the strong coupling $\alpha_\perp$ appears to be $\alpha_\perp = 1$, which implies that $\zeta \to 0$ and $z \to 2$ in high dimensions. Similar $1/t$ corrections to the growth rate arise generically [28] in mean field approximations [7, 29] to growth processes.

(iv) $\lambda = 0$. Recently a class of growth models was introduced [6, 7, 13, 30, 31] for which the coefficient λ (equation (5)) changes sign as a temperature-like parameter κ is varied. According to our result (2), monitoring the finite-size corrections for different κ should provide a simple and efficient way of determining the transition point κ_c where $\lambda(\kappa_c) = 0$. We illustrate this by simulations of a solid-on-solid model due to Amar and Family [30]. In figure 3 we plot the amplitude of the spatial finite-size correction as a function of κ for one- and two-dimensional substrates. We estimate the critical parameter value to be $\kappa_c(d=1) \simeq 0.951$ aand $\kappa_c(d=2) \simeq 0.557$. A change in the sign of $\Delta G(L)$ was also observed by Gates and Westcott [6] in an exactly solved one-dimensional model [7, 31].

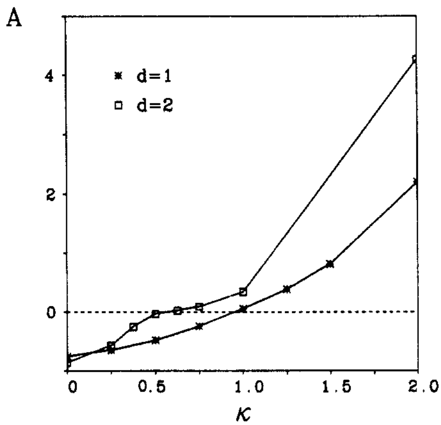

Figure 3. Amplitude of the spatial finite-size correction for the Amar–Family model [30]. The amplitude A was determined by fitting the growth rate to the form $G = G_0(1 - A/L^{\alpha_\parallel})$ for $L = 16 - 1024$ ($d = 1$) and $L = 16 - 512$ ($d = 2$). For the correction exponent the values $\alpha_\parallel = 1$ ($d = 1$) and $\alpha_\parallel = 1.25$ ($d = 2$) were used.

At $\kappa = \kappa_c$ the finite-size corrections are determined by higher-order terms in the macroscopic inclination dependent growth rate $G_0(\boldsymbol{u})$. Since such nonlinearities are irrelevant for the large-scale behaviour of surface fluctuations, statistical averages can be evaluated with the Gaussian correlations of the linearized KPZ equation. This implies that $\boldsymbol{q}^2\langle|h_{\boldsymbol{q}}|^2\rangle \simeq L^{-d}\tilde{\chi}$ for small q, with some constant $\tilde{\chi}$, and higher moments of $(\nabla h)^2$ factorize as $\langle[(\nabla h)^2]^n\rangle = B_n\langle(\nabla h)^2\rangle^n$ with $B_n = \Pi_{k=1}^{n-1}(2k+1)$. To model a situation where $G_0(\boldsymbol{u}) \approx G_0(0) + \lambda_4|\boldsymbol{u}|^4$ for small $\boldsymbol{u}$, we add a term $\lambda_4[(\nabla h)^2]^2$ to the right-hand side of the KPZ equation (3). The full growth rate $G_0(\boldsymbol{u})$ is computed using the factorization property, and it is found that the term proportional to $\boldsymbol{u}^2$ vanishes if the parameter in the KPZ equation are adjusted to $\lambda = -12\tilde{\chi}\lambda_4$. The finite-size correction to $\langle[(\nabla h)^2]^2\rangle$ is then evaluated with this choice of parameters, which ensures that the physical growth rate has $\lambda = 0$. This gives a correction $\Delta G \sim L^{-2d}$. In general, if the lowest-order nonlinearity in $G_0(\boldsymbol{u})$ is $\lambda_{2n}|\boldsymbol{u}|^{2n}$, the spatial finite-size correction is proportional to $(-1)^n\lambda_{2n}L^{-nd}$, so $\alpha_\parallel = nd$ and $\alpha_\perp = \alpha_\parallel/z = nd/2$.

(v) *Faceting transitions* are associated with a non-analytic variation of the growth rate with surface inclination [32], $G_0(\boldsymbol{u}) - G_0(\boldsymbol{0}) \sim |\boldsymbol{u}|^\theta$ with $\theta < 2$, corresponding formally to $\lambda = \infty$. Such cases are covered by the following simple scaling argument, which also reproduces the previous results for the exponents. Transverse excursions on a lateral length scale $\xi_\parallel$ are of the order $\xi_\perp \sim \xi_\parallel^\zeta$. Hence the inclination fluctuations scale as $\xi_\perp/\xi_\parallel \sim \xi_\parallel^{-(1-\zeta)}$, and the corresponding contribution to the growth rate is of the order $\xi_\parallel^{-\theta(1-\zeta)}$. In a finite system there are contributions up to $\xi_\parallel \sim L$ or $\xi_\parallel \sim t^{1/z}$ depending on which length scale is smaller. Thus we conclude thaat

$$\alpha_\parallel = \theta(1-\zeta) \qquad \alpha_\perp = \theta(1-\zeta)/z. \tag{16}$$

At the critical point of percolation-driven faceting [32] θ is the directed percolation anisotropy exponent, and it can be shown [33] that $\zeta = 0$, $z = \theta$. Equation (16) then reduces to $\alpha_\parallel = \theta$, $\alpha_\perp = 1$ in agreement with previous results for the growth rate [33].

(vi) *Deterministic growth.* In the absence of noise (ν, $\eta \to 0$) the KPZ equation (3) describes the smoothing of a rough substrate, which is modelled by an ensemble of random initial conditions [34, 35]. Asymptotically the growth rate approaches its bare value g_b. The exponent $\alpha_\perp$ of the transient power law correction is still given by (9); however now ζ denotes the effective [7] substrate wandering exponent and z is determined through $z = 2 - \zeta$ [34]. There are no spatial finite-size corrections, since the surface is completely flat for $t \gg L^z$. For the deterministic KPZ equation with generalized nonlinearity [34] equation (16) holds, as can be checked for an exactly solved case [34].

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References

[1] Gilner G H 1980 *J. Cryst. Growth* **49** 465
[2] Chernov A A 1984 *Modern Crystallography III: Crystal Growth* (Springer Series in Solid State Sciences **36**) (Springer: Berlin)
[3] Lauritzen J I 1973 *J. Appl. Phys.* **44** 4353
[4] Frank F C 1974 *J. Cryst. Growth* **22** 233
[5] Goldenfeld N 1984 *J. Phys. A: Math. Gen.* **17** 2807
[6] Gates D J and Westcott M 1988 *Proc. R. Soc.* A **416** 463
[7] Krug J and Spohn H 1990 *Solids Far From Equilibrium: Growth, Morphology and Defects* ed C Godrèche (Cambridge: Cambridge University Press)
[8] Kardar M, Parisi G and Zhang Y C 1986 *Phys. Rev. Lett.* **56** 889
Medina E, Hwa T, Kardar M and Zhang Y C 1989 *Phys. Rev.* A **39** 3053
[9] van Saarloos W and Gilmer G H 1986 *Phys. Rev.* B **33** 4927
[10] Plischke M and Ràcz Z 1985 *Phys. Rev.* A **32** 3825
[11] Wolf D E and Tang LH *Preprint* KFA Jülich
[12] Krug J 1989 *J. Phys. A: Math. Gen.* **22** L769
[13] Krug J and Spohn H 1990 *Phys. Rev. Lett.* **64** 2332
[14] Lipowsky R 1988 *Random Fluctuations and Pattern Growth* ed H E Stanley and N Ostrowsky (*NATO ASI Series* **E157**) (Dordrecht: Kluwer)
[15] Forrest B M and Tang L-H 1990 *Phys. Rev. Lett.* **64** 1405
Renz W, private communication
[16] Krug J 1987 *Phys. Rev.* A **36** 5465
[17] Meakin P 1985 *Multiple Scattering of Waves in Random Media and Random Rough Surfaces* ed V V Varadan and V K Varadan (University Park, PA: Pennsylvania State University)
[18] Meakin P, Ramanlal, Sander L M and Ball R C 1986 *Phys. Rev.* A **34** 5091

[19] Kim J M and Kosterlitz J M 1989 *Phys. Rev. Lett.* **62** 2289
[20] Meakin P and Jullien R 1990 *Phys. Rev.* A **41** 983
[21] Bensimon D, Shraiman B and Liang S 1984 *Phys. Lett.* **102A** 238
Liang S and Kadanoff L P 1985 *Phys. Rev.* A **31** 2628
Joag P S, Limaye A V and Amritkar R E 1987 *Phys. Rev.* A **36** 3395
[22] Kertész J and Wolf D E 1988 *J. Phys. A: Math. Gen.* **21** 747
[23] Forster D, Nelson D R and Stephen M J 1977 *Phys. Rev.* A **16** 732
Huse D A, Henley C L and Fisher D S 1985 *Phys. Rev. Lett.* **55** 2924
[24] Krug J and Spohn H 1989 *Europhys. Lett.* **8** 219
[25] Krug J unpublished
[26] Kardar M and Zhang Y C 1987 *Phys. Rev. Lett.* **58** 2087
Cook J and Derrida B 1989 *J. Stat. Phys.* **57** 89
[27] Derrida B and Spohn H 1988 *J. Stat. Phys.* **51** 817
[28] van Saarloos W 1989 *Phys. Rev.* A **39** 6367
[29] Krug J and Meakin P *Preprint* University of Munich
[30] Amar J G and Family F 1990 *Phys. Rev. Lett.* **64** 543
[31] Brandstetter H *Diploma thesis* University of Munich (in preparation)
[32] Krug J, Kertész J and Wolf D E 1990 *Europhys. Lett.* **12** 113
[33] Kertész J and Wolf D E 1989 *Phys. Rev. Lett.* **62** 2571
[34] Krug J and Spohn H 1988 *Phys. Rev.* A **38** 4271
[35] Tang C, Alexander S and Bruinsma R 1990 *Phys. Rev. Lett.* **64** 772